DOCUMENT RESUME

ED 191 673 SE 031 743

TITLE Program of Studies - Mathematics, K-8.
INSTITUTION Montgomery County Public Schools, Rockville, Md. Dept. of Instructional Planning and Development.
PUB DATE 79
NOTE 38p.; For related document, see SE 031 744.

EDRS PRICE MF01/PC02 Plus Postage.
DESCRIPTORS Cooperative Programs; Coordination; *Course Objectives; *Educational Objectives; Elementary Education; *Elementary School Mathematics; Lesson Plans; *Mathematics Curriculum; *Mathematics Education; Mathematics Instruction; *Program Descriptions; Units of Study

ABSTRACT

The mathematics education goals for Montgomery County Public Schools are listed in this public policy statement. The basic skills in mathematics that the school system wants each child to attain are the ability to perform computations, solve common problems of mathematics and logic, and understand the structure of mathematics so it can be a useful tool in daily living. A chart of the possible patterns of study in grades seven through twelve is provided and eighteen levels of mathematics objectives for grades kindergarten through eight are listed. Descriptions of alternative seventh- and eighth-grade courses in mathematics are included at the end of this publication. This document is the first of two booklets describing the entire school system's mathematics program. (MP)

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SPRING 1977

PROGRAM OF STUDIES

MATHEMATICS, K-8

Department of Instructional Planning and Development
Montgomery County Public Schools
Rockville, Maryland

Spring 1979

SEP 2 1980

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Rockville, Maryland

The Goals of Education

Adopted by the Board of Education — February 12, 1973

The goals of education for the Montgomery County Public Schools are set forth in this statement of public policy to guide the school system in developing, implementing, and improving educational programs for its students. This statement outlines those goals considered by the citizens of the county as most important for the education of their children in public schools and for which the citizens will hold the school system responsible and accountable.

The school system has the primary responsibility for some of these goals. For others, it shares that responsibility with the home and other community institutions.

The extent to which these goals can be realized will depend upon the endorsement by and continuing commitment of the school system staff, students, parents, and the community. With broad endorsement and wide support, the Montgomery County Public Schools will continue its development to serve its students and community in the most effective way possible.

Education is a process that encompasses the total experiences of each individual throughout a lifetime of formal and informal learning. The school program, as a keystone in this process, should provide opportunities and encouragement for students to acquire knowledge, to explore ideas, to ask questions, and to seek answers that will lead to sound and useful conclusions.

The probability of success motivates students to learn. Each school must create a program and maintain a climate in which every student has opportunities for success. Each individual is unique, and the school shall encourage that uniqueness. The school should help the student understand his [/her] values and the values of others.

The school program, while developing the skills of learning, should be based on the study of broad human concerns, flexible enough to deal with changing concerns and at the same time related to the needs, interests, and concerns of each student. The program should offer opportunities for decision-making. It should help the student develop the capacity to learn throughout his [/her] lifetime, to respond to and understand other human beings, and to accept full responsibility for the results of his [/her] actions.

Therefore, the Montgomery County Public Schools dedicates itself to provide the opportunity, encouragement, and guidance to make it possible for every child to attain the following goals of education:

ACADEMIC SKILLS

The fundamental responsibility of the school is to develop programs that enable each child to acquire those skills basic to all learning. The achievement of all other goals depends upon the success of the school in ensuring that each student, according to his [/her] own abilities, attain the following basic skills:

- **Reading:** The ability to read and comprehend written material and relate it to other knowledge
- **Composition:** The ability to write with precision, clarity, and acceptable usage, whether to inform, inspire, or persuade
- **Listening and Speaking:** The ability to listen attentively and with understanding and to speak with confidence and effectiveness, whether from written material or extemporaneously
- **Mathematics:** The ability to perform computations, to solve common problems of mathematics and logic, and to understand the structure of mathematics so it can be a useful tool in daily living
- **Study:** The development of basic study skills so that he [/she] may acquire knowledge efficiently
- **The Arts:** The development of some of the basic disciplines and skills in the performing and creative arts to be used throughout his [/her] life for communication, expression, and enjoyment
- **Observation:** The ability to identify and differentiate elements of the world around him [/her] as they are useful in personal, academic, and artistic pursuits

PHYSICAL DEVELOPMENT

Each person matures physically at a different rate and possesses differing capabilities. The school has the obligation to help each student:

- understand the biological functioning of his [/her] body
- make the best both of physical talents and limitations
- develop good health habits, skills, and interests to maintain his [/her] body in optimum condition throughout his [/her] lifetime

INTELLECTUAL DEVELOPMENT

In addition to acquiring academic skills, each individual should develop his [/her] intellectual capabilities to the fullest extent possible. Therefore, the school will encourage each pupil:

- to think creatively
- to reason logically
- to apply knowledge usefully
- to deal with abstract concepts
- to solve problems

THE INDIVIDUAL AND SOCIETY

Every person must learn to live in a society. The school must help each student develop an understanding of [people] and of how the individual depends upon others and they on him [/her]. This requires that each student gain:

- knowledge of himself [/herself] and the characteristics, needs, and desires he [/she] shares with others
- sensitivity to others and their ideas, and the ability to act responsibly in various situations
- the ability to function productively as a member of a group
- familiarity with the legal, moral, ethical, and cultural heritages of his [/her] and other societies
- knowledge of the various political systems and philosophies of the world

SCIENTIFIC UNDERSTANDING

[We] must exist in the natural world. The school must help each student understand nature's impact on [us] and [our] impact on nature. This requires that each student gain:

- knowledge of natural phenomena and their effect on [people]
- understanding of scientific advances and their part in modern technology
- understanding of the scientific method
- appreciation of the ways in which the application of scientific principles can improve the quality of life while preserving the natural order

AESTHETIC EXPRESSION

[We], to realize [our] full potential, must be able to sense and appreciate beauty in the world around [us], whether created by nature or by [people]. The school must help each student gain:

- knowledge of the nature of the creative performing arts
- experience with a wide variety of art forms
- a perspective for developing his [/her] own aesthetic criteria and tastes
- understanding of the contribution of the arts to human communication

CAREER DEVELOPMENT

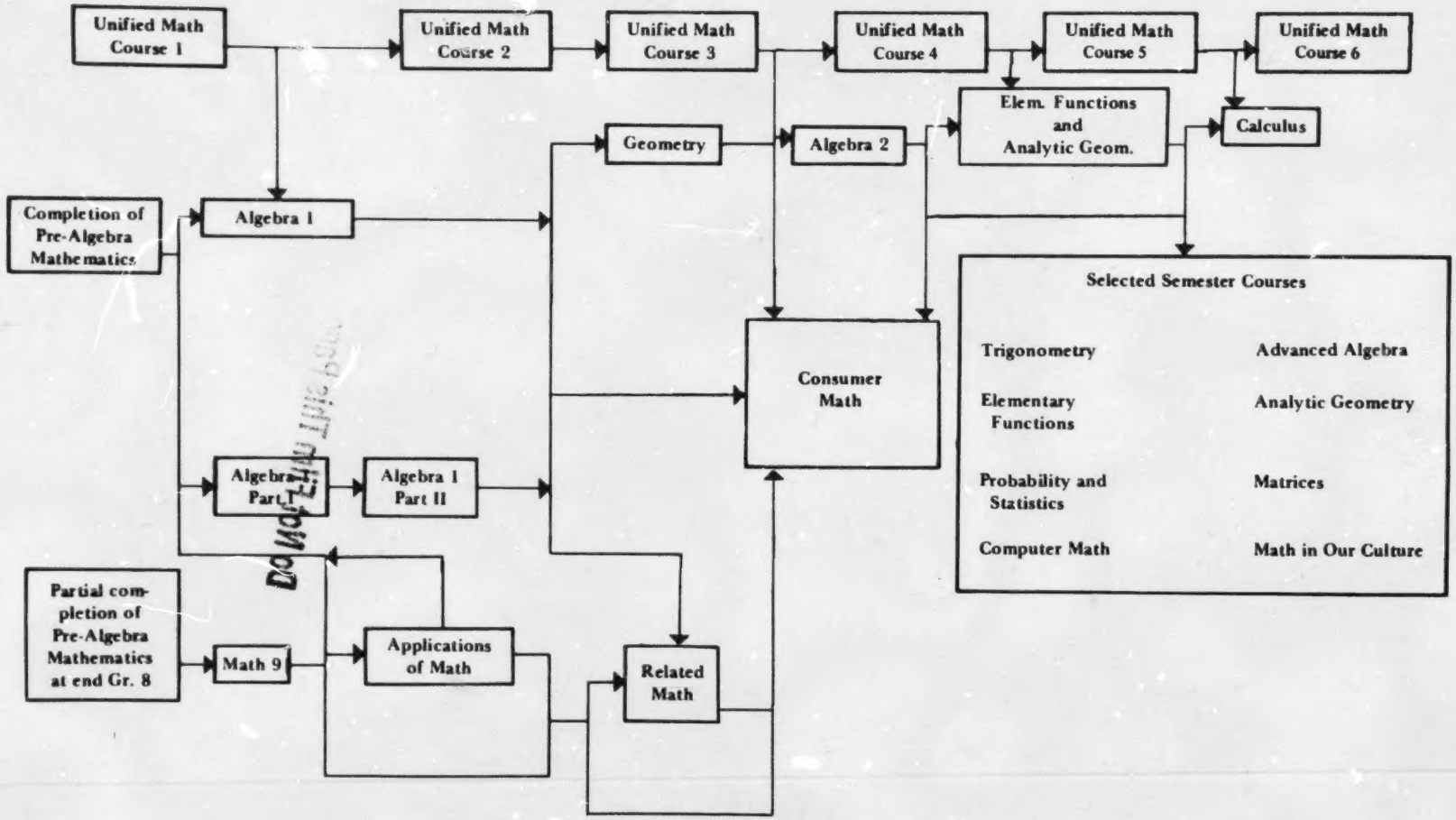
Productive and satisfying work enriches [our lives]; and with increasing leisure time, many people will use various occupational skills for avocational purposes as well. The school must help each student gain:

- knowledge and appreciation of the wide variety and interrelationships of occupations in modern society
- opportunities to explore potential occupations in relation to personal aptitudes and interests, unrestricted by stereotypes of sex, race, or socio-economic level
- the knowledge, skills, and abilities that enable him [/her] to secure satisfying employment, embark upon further training and education in a chosen career field, and adapt occupational talents to changing job demands and opportunities

COMMITMENTS

The Board of Education and the staff of the Montgomery County Public Schools must create and maintain an educational environment that promotes the attainment of these goals by all students. Toward this end, therefore, the public schools make the following commitments to the citizens of Montgomery County:

- *To attract and retain the most qualified and best trained staff possible through dynamic programs of recruitment, supervision, and continuing in-service training and staff development opportunities*
- *To develop and implement a comprehensive and flexible instructional program to achieve the aforementioned goals and to provide the facilities, materials, and equipment needed to enhance the effectiveness of the program*
- *To evaluate the effectiveness of the instructional program continuously, and regularly report the findings to students, parents, staff, and the community*
- *To adopt new and different approaches when it is determined that they will contribute more effectively to the fulfillment of the goals*
- *To encourage a continuing dialogue with the entire community, making every effort to communicate needs and achievements, and to be responsive to the needs and aspirations of the school community*
- *To utilize facilities and staff, in cooperation with other agencies, in the development of preschool, community school, and adult education programs that will meet the continuing educational needs of all citizens*
- *To provide an accountability procedure for informing county citizens of the objectives and costs of their school system and of reporting periodically on the educational investments made with their tax dollars*



MATHEMATICS PROGRAM PATTERNS CHART

MATHEMATICS, GRADES K-8

POINT OF VIEW

A provocative activity which teachers often use with pupils at various levels is that of trying to imagine what a world without numbers would be like. Such a world is difficult to imagine. The idea of number continues to play an important role in virtually all aspects of our world; and mathematics, therefore, constitutes a program of considerable importance in the schools.

As a discipline, mathematics is truly the art and science of abstraction. Characteristics of the physical world are converted into abstract ideas and symbols; these are then manipulated through mathematical operations to produce information and theorems about less easily observed aspects of the world. Recent evidence supports the contention that children's experiences with concrete materials are vital to later conceptual development. The school program thus proceeds from the concrete to the abstract.

The concepts of mathematics acquire greater meaning when they can be applied to the world in which we live. Because the variety and extent of mathematical applications have grown so rapidly in recent years, it is impossible for any one person to be conversant with the entire field. The school program must therefore be developed so that mathematical applications are selected and presented as efficiently as possible and with the intent of challenging pupils at all levels to see mathematics as an independent discipline as well as a tool for the advancement of other disciplines.

The Montgomery County mathematics program is designed and implemented to take into account the logical and relatively sequential nature of mathematics. Equally important, too, is the realization that the rate at which individual students learn mathematics varies significantly. The mathematics program is structured to encourage various approaches which allow students to progress at their individual rates.

The pre-algebra objectives range over six areas of mathematics and are arranged according to 14 different levels of achievement. Assessment measures have been constructed for each objective so that individual progress can be measured in a variety of categories. Enrichment activities are available for both the able student and the student who needs reinforcement.

Several options are available to the student who has completed the pre-algebra objectives. Differentiated paths through a variety of courses are available to the student, as can be seen in the Mathematics Program Patterns Chart on page v. Each student has available a sequence of courses which can be suited to his/her abilities.

Computer literacy is addressed at several levels in the mathematics program; career information is incorporated as appropriate throughout. Consumer applications are taught as mathematical skills are developed; the mathematics of consumerism is further emphasized in an elective senior high course.

In general terms, the instructional program in mathematics should help each student to:

Develop basic skills in using the vocabulary and symbols of mathematics

Develop skills in recognizing common geometric shapes

Develop basic skills in computing

Develop basic skills in working with geometric shapes

Develop basic skills in measuring, graphing, and using tables and charts

Develop understanding of the vocabulary and symbols of mathematics

Develop understandings necessary for translating among mathematical symbols, words, and the physical world

Develop concepts related to common geometric shapes

Develop understanding of computation

Develop understanding of measurement

Develop an understanding of basic principles related to the structure of mathematics

Develop understanding and basic skills in problem solving

Apply the principles of mathematical reasoning to the solution of problems

Appreciate the significance of mathematics in daily living and its contribution to our cultural heritage

Use mathematics as needed in daily living

GENERAL INFORMATION

The objectives of the Math program are compatible with the philosophy and commitment of the MCPS Goals of Education* and are specifically aligned with the Academic Skills area. The goal in this area states that the school must ensure that each student, according to his/her unique capabilities, attains the following basic skills:

- The ability to perform computations, to solve common problems of mathematics and logic, and to understand the structure of mathematics so it can be a useful tool in daily living

- The development of basic study skills so that he [/she] may acquire knowledge efficiently

The county mathematics program is generally organized into two levels. Elementary and pre-algebra mathematics begins with the kindergarten program and continues until the student has completed those mathematical objectives which are prerequisite to the study of algebra or applications of mathematics. A student may enter Algebra 1 upon demonstrating that the pre-algebra objectives have been attained, any time beginning in Grade 7. Mathematically and verbally able sixth grade students in certain schools may be selected to enter the Unified Mathematics program in Grade 7. The sequence of courses following Algebra 1 in the MCPS mathematics program calls for Geometry, followed by Algebra 2. This sequence was chosen to satisfy college admission requirements and textbook content, and to provide consistency in offerings for students moving from one school to another. Beginning with Grade 9, the program provides an increasing number of electives outside the Unified Mathematics and Algebra-Geometry sequences, as the student progresses further in senior high school. At all levels, content and learning behaviors are introduced in a spiral manner, proceeding at gradually increasing levels of difficulty across a variety of topics.

The Maryland State Department of Education has constructed a Grade 7 mathematics test to be given annually statewide to all Grade 7 students. This test measures achievement on specified basic skills objectives. The Functional Mathematics Test is mandated by the State of Maryland for all Grade 9 students; students who do not meet passing requirements on this test will be tested again in Grade 10.

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*Adopted February 12, 1973 by the Board of Education of Montgomery County, Maryland

Approved textbooks for the mathematics K-8 program are listed in the MCPS *Elementary Textbook List, Secondary Textbook List.* Approved films are listed in the MCPS *Film Catalog.*

DESCRIPTION OF PROGRAM — MATHEMATICS

The content of the elementary and pre-algebra mathematics program is organized to develop skills and understandings which will enable students to enter successfully into the study of algebra, should they choose to do so. Experiences dealing with physical objects provide the initial setting which helps the child see numbers as one way of describing the world in which he/she lives. Whole numbers, fractional numbers, geometry, and measurement are often introduced and explored with concrete objects in such a way as to facilitate later mastery on an abstract level. Later the rational numbers and higher level applications are studied as more abstract concepts are needed. Since mathematics is cumulative, ample opportunities are provided to maintain as well as reinforce skills and understandings.

The elementary and pre-algebra mathematics program is designed to encourage pupils to progress individually toward goals which are performance based. These performance objectives are organized across a series of levels denoted alphabetically which extend from kindergarten through that point at which the pupil is ready to enter into the study of algebra or some other mathematics course of his/her choice. Two alphabetic levels are equivalent to approximately one year of progress. Teachers are encouraged to provide opportunities for individual pupils to progress at an appropriate rate. Individualized instruction, small group instruction, and large group instruction may all be employed in the program. The program is also designed to facilitate the keeping of records on the progress of individual pupils. These records can then help teachers provide continuity which extends across grade levels. The program encourages long-range planning and flexible class grouping. It also reflects the fact that children learn better and remember longer when instruction spirals across the various categories, returning to each several times during the course of a year.

The mathematics objectives, K-8, comprise a continuous program and are organized by content topics and categories to facilitate planning across levels. Objectives through Level F describe the primary or K-2 program; objectives from Level G through Level L describe the Grades 3-5 program; objectives from Level M through R describe the 6-8 program. The objectives listed in this document are the basis for the Instructional System in Mathematics (ISM) currently operating in some elementary, middle, and junior high schools. Schools not on ISM are using the objectives as printed in the MCPS Bulletin 285, *Elementary and Pre-Algebra Mathematics Objectives.* Instructional system mathematics guides have been developed and are provided to all teachers as they enter the instructional system. ISM is projected for all schools by FY 1981.

LEVEL A

Topic: Whole Numbers
Category: Numeration

Matches two sets one-to-one (same size or not)

Chooses the larger of two sets

Chooses the smaller of two sets

Equalizes a smaller set to a larger set

Equalizes a larger set to a smaller set

Topic: Geometry and Measurement
Category: Length

Compares and orders by length (long, short, tall, taller, etc.)

Category: Geometric Figures

Sorts objects by likenesses in both size and shape

LEVEL B

Topic: Whole Numbers
Category: Numeration

Says number names (1-9)

Names the number of a set (1-9)

Makes sets with objects (1-9)

Reads numerals (1-9)

Matches numerals to sets (1-9)

Constructs sets with one more

Constructs sets with one less

Identifies elements before, after, or between in a sequence

Topic: Geometry and Measurement
Category: Time and Temperature

Compares and orders by time (before, after, earlier)

Category: Length

Equalizes given lengths (adding or taking away)

Category: Capacity

Identifies a container as full, empty, or partly full

Category: Geometric Figures

Sorts objects by likenesses in shape but not size

Identifies and names rectangular, circular, and triangular shapes

Copies a given pattern

Category: Statistical Graphs and Tables

Constructs a pictograph with a group

Describes comparisons from a pictograph constructed by the group

Category: Coordinate Graphs

Describes locations within the immediate environment

Topic: Fractions
Category: Common Fractions

Identifies and demonstrates one-half of a region or object

LEVEL C

Topic: Whole Numbers
Category: Numeration

Orders sets

Orders numerals (1-9)

Uses zero to name a set with no members

Writes numerals 0-9

Demonstrates what is needed to make two unequal sets of objects equal in number (1-9)

Category: Place Value

Groups objects into sets of ten

Demonstrates a teen number as a set of ten and 1-9 ones

Names sets of tens to 90

Category: Addition

Puts two sets of objects together and names the number of the new set

Reads and demonstrates with objects addition sentences (sums to 10)

Category: Subtraction

Takes elements from a set of objects and names the number of the set removed and of the remaining set

Reads and demonstrates with objects subtraction sentences (sums to 10)

Topic: Geometry and Measurement
Category: Length

Measures length, using non-standard units

Category: Weight and Mass

Compares weights on a primary balance

Category: Geometric Figures

Identifies and names cube, sphere, cone

Chooses a 3-D object that has a face of a given shape (□ ○ △)

Topic: Other
Category: Number Sentences

Demonstrates the meaning of the symbols = , > , and < by comparing sets of concrete objects (numbers 0-9)

LEVEL D

Topic: Whole Numbers
Category: Numeration

Reads and writes two-place numerals

Names missing numbers 0-100, before, after, or between given numbers (counting by ones)

Names missing numbers 0-100, before, after, or between given numbers (counting by tens)

Category: Place Value

Demonstrates with objects and names the number of tens and ones in numerals through two places

Topic: Geometry and Measurement
Category: Money

Identifies and states value of a penny, nickel, and dime

Uses symbol for cent (¢)

Category: Time and Temperature

Identifies time on a clock (hours)

Identifies day, week, and month on a calendar

Category: Capacity

Measures the capacity of a container with objects by counting

Category: Geometric Figures

Classifies paths (straight or not straight, closed or not closed)

Continues a repeating geometric pattern

Category: Statistical Graphs and Tables

Constructs a bar graph with a group

Interprets a bar graph constructed by the group

Topic: Fractions
Category: Common Fractions

Distinguishes among one-half, one-third, and one-fourth of a region or object

Topic: Other
Category: Number Sentences

Reads orally number sentences using the symbols = , > , and < (sums to 10)

LEVEL E

Topic: Whole Numbers
Category: Numeration

Reads and writes three-place numerals

Names objects with ordinal number names, and identifies objects given ordinal names to tenth

Names missing numbers in a sequence (before, after, or between given numbers, up to 1,000, counting by ones)

Names missing numbers in a sequence (before, after, or between given numbers, up to 1,000, counting by tens)

Names missing numbers in a sequence (before, after, or between given numbers, up to 1,000, counting by fives)

Names missing numbers in a sequence (before, after, or between given numbers, up to 1,000, counting by twos)

Category: Place Value

Names 10 tens as a set of 100

Names sets of hundreds to 900

Category: Addition

Adds tens (sums to 90)

Increases two-place numbers by 10

Adds two-place numbers without regrouping

Category: Subtraction

Subtracts tens (sums to 90)

Decreases two-place numbers by 10

Subtracts two-place numbers without regrouping

Category: Basic Facts

Names addition facts (sums to 10)

Names subtraction facts (sums to 10)

Topic: Geometry and Measurement

Category: Time and Temperature

Identifies days with ordinal number names to thirty-first

Category: Weight and Mass

Weighs with nonstandard units on a balance scale

Category: Geometric Figures

Demonstrates line symmetry in a given shape (by folding)

Identifies faces, edges, and corners of 3-D objects

Identifies objects by two or three properties

Category: Statistical Graphs and Tables

Constructs a simple pictograph

Interprets a simple pictograph

Topic: Other
Category: Number Sentences

Writes number sentences using the symbols =, >, and <

Chooses the number sentence which describes a problem situation (sums to 10)

LEVEL F

Topic: Whole Numbers
Category: Place Value

Demonstrates with objects and names the number of hundreds, tens, and ones in numerals through three places

Expands three-place numerals

Category: Addition

Demonstrates the principle of regrouping in two-place numbers

Category: Subtraction

Demonstrates the principle of regrouping in two-place numbers

Category: Multiplication

Combines given equivalent sets and names the number of the new set

Category: Division

Separates a set of objects into subsets of a given size, and names the number of subsets

Separates a set of objects into a given number of subsets, and names the number in each set

Topic: Geometry and Measurement
Category: Money

Identifies and states value in cents of a penny, nickel, dime, quarter, half dollar, and dollar

Category: Time and Temperature

States time after the hour on a clock (five-minute intervals)

States relationships: minutes to hour, days to week, weeks to month, months to year

Category: Length

Measures and uses symbols for meters (m), decimeters (dm), and centimeters (cm)

Category: Geometric Figures

Constructs and describes a repeating pattern of geometric shapes

Category: Statistical Graphs and Tables

Constructs a simple bar graph

Interprets a bar graph

Category: Coordinate Graphs

Shows locations on grids as blocks over and up

Topic: Fractions
Category: Common Fractions

Identifies one-tenth of a region or object

Identifies parts of a region (halves, thirds, fourths, tenths)

Topic: Other
Category: Number Sentences

Solves number sentences with a missing number in any position (addition and subtraction facts)

Category: Estimation and Rounding

Rounds two-place numbers to the nearer 10

LEVEL G

Topic: Whole Numbers
Category: Place Value

Names 10 hundreds as 1,000

Demonstrates with objects four-place numerals

Names 10 thousands as 10,000

Category: Addition

Adds two-place numbers with regrouping

Adds hundreds (sums to 900)

Adds three-place numbers without regrouping

Demonstrates the regrouping of three-place numerals

Category: Subtraction

Subtracts two-place numbers with regrouping

Subtracts hundreds (sums to 900)

Subtracts three-place numbers without regrouping

Demonstrates the regrouping of three-place numerals

Category: Basic Facts

Names addition facts (sums to 18)

Names subtraction facts (sums to 18)

Topic: Geometry and Measurement
Category: Length

Describes uses of the kilometer (km)

Renames kilometers to meters

Category: Geometric Figures

Identifies and describes shapes having line symmetry

Identifies congruent shapes by matching

Category: Statistical Graphs and Tables

Names information from a table

Interprets a simple circle graph

Topic: Other
Category: Estimation and Rounding

Rounds three-place numbers to the nearer 10 or 100

LEVEL H

Topic: Whole Numbers
Category: Addition

Adds three-place numbers with regrouping

Category: Subtraction

Subtracts three-place numbers with regrouping

Category: Multiplication

Demonstrates with objects the multiplication facts of 1, 2, 3, 4, 5

Multiplies multiples of 10, 100, and 1,000 by one-digit numbers (1, 2, 3, 4, 5)

Multiplies two- and three-place factors by 2, 3, 4, 5 with no regrouping

Multiplies two- and three-place factors by one-digit factors (2, 3, 4, 5) with regrouping

Category: Division

Demonstrates with objects the division facts of 1, 2, 3, 4, 5

Divides multiples of 10, 100, and 1,000 by one-digit divisors (1, 2, 3, 4, 5) (no remainders)

Divides two-place dividends by one-digit divisors (1, 2, 3, 4, 5) (with or without remainders)

Category: Basic Facts

Names multiplication facts of 0, 1, 2, 3, 4, 5

Names division facts of 1, 2, 3, 4, 5

Topic: Geometry and Measurement
Category: Money

Selects coins to equal a given value

Makes change (adding up)

Uses dollar symbol ($)

Adds and subtracts money

Category: Time and Temperature

Reads time notation (2:35 as "two thirty-five")

Reads and writes time (A.M., P.M., 2:35)

Category: Weight and Mass

Weighs and records grams (g) or kilograms (kg)

Category: Capacity

Measures and records capacity in liters (L) and milliliters (mL)

Category: Statistical Graphs and Tables

Tabulates data into a table

Constructs a pictograph from data gathered and tabulated

Topic: Fractions
Category: Common Fractions

Demonstrates one-half, one-third, and one-fourth of sets

States the meaning of each term of a common fraction

Reads and writes symbols for unit fractions using regions (1/2, 1/3, 1/4, 1/10)

Names and writes common fractions <1 shown on a given region (denominators of 2, 3, 4, 6, 8, 10)

LEVEL I

Topic: Whole Numbers
Category: Numeration

Reads and writes four- or five-place numerals

Category: Place Value

States the place and value of any digit in four- or five-place numerals

Expands four- and five-place numerals

Identifies the value of the first position to the right of the decimal point as one-tenth of the ones place

Identifies the value of the second position to the right of the decimal point as one-tenth of the tenths place and one-hundredth of the ones place

Category: Addition

Adds through four-place numbers with regrouping

Category: Subtraction

Subtracts through four-place numbers with regrouping

Category: Multiplication

Multiplies two- and three-place factors by one-digit factors (6, 7, 8, 9) with regrouping

Category: Division

Divides multiples of 10 greater than 100 by one-digit divisors (6, 7, 8, 9) (no remainders)

Topic: Geometry and Measurement
Category: Length

Measures in millimeters (mm)

Renames meters through millimeters

Category: Area and Volume

Distinguishes between a plane shape and its region

Identifies shapes which can fit together to cover a surface

Category: Statistical Graphs and Tables

Constructs a bar graph from data gathered and tabulated

Category: Coordinate Graphs

Identifies the location for a given ordered pair, and writes the ordered pair for a given location on a coordinate graph (first quadrant)

Graphs pairs of addends for a given sum on a coordinate graph (first quadrant)

Topic: Fractions
Category: Common Fractions

Identifies and demonstrates two-thirds and three-fourths of sets

Names and writes common fractions < 1 shown on a given set (denominators of 2, 3, 4, 6, 8, 10)

Identifies fractional names for one

Identifies the *numerator* and *denominator* and states the meaning of any given common fraction

Category: Decimal Fractions

Demonstrates decimal fractions through hundredths

Reads and writes decimal fractions through hundredths

Writes common fractions for decimal fractions and decimal fractions for common fractions (tenths, hundredths)

Topic: Other
Category: Factors and Multiples

Identifies factors in a multiplication or division number sentence

Names multiples of whole numbers to 9

Category: Estimation and Rounding

Rounds four-place numbers to the nearer 10, 100, or 1,000

Estimates sums or differences

LEVEL J

Topic: Whole Numbers
Category: Division

Divides three-place numbers by one-digit divisors with two-place quotients (with or without remainders)

Divides three-place numbers by one-digit divisors with three-place quotients (with or without remainders)

Category: Basic Facts

Names multiplication facts of 6, 7, 8, 9

Names division facts of 6, 7, 8, 9

Topic: Geometry and Measurement
Category: Money

Multiplies, using money

Category: Time and Temperature

States time on clock to minute (2:43)

States the relationship of seconds to minutes and minutes to seconds

Category: Length

Measures perimeters of plane figures

Category: Area and Volume

Finds and records areas of plane regions by counting square units

Category: Geometric Figures

Classifies and identifies polygons with 4, 5, 6, 8 line segments

Category: Statistical Graphs

Interprets information from a line graph

Constructs graphs using decimal fractions

Topic: Fractions
Category: Commmon Fractions

Identifies equivalent names for fractions ≤ 1

Demonstrates the meaning of any common fraction ≥ 1

Writes fractions >1 as mixed numerals and mixed numerals as fractions

Category: Decimal Fractions

Adds and subtracts decimal fractions through hundredths with regrouping

Orders decimal fractions through hundredths

Topic: Other
Category: Number Sentences

Solves number sentences with a missing number in any position (multiplication and division facts)

Category: Factors and Multiples

Names common multiples of two whole numbers to 16

Names all factors of whole numbers to 50

Names common factors of two whole numbers to 50

Names prime numbers to 50

Names the greatest common factor of two or more whole numbers to 50

Category: Positive and Negative Numbers

Uses positive and negative numbers in a game situation

LEVEL K

Topic: Whole Numbers
Category: Addition

Adds with regrouping

Category: Subtraction

Subtracts with regrouping

Category: Multiplication

Multiplies two- and three-place factors by 10 and multiples of 10 (including 100)

Category: Division

Divides three- and four-place dividends by multiples of 10 with two- and three-place quotients (with or without remainders)

Topic: Geometry and Measurement
Category: Time and Temperature

Adds and subtracts, using time

Category: Area and Volume

Computes the area of rectangles (l x w =A)

Uses exponential symbols for metric areas units (cm^2)

Category: Geometric Figures

Identifies the reflection of a figure

Draws the reflection of a given figure

Locates the lines of symmetry in shapes

Category: Statistical Graphs and Tables

Determines averages for given data

Topic: Fractions
Category: Common Fractions

States fractions and mixed numerals in simplest form (i.e., lowest terms)

Orders pairs of fractional numbers, using >, <, =

Demonstrates with objects the addition of fractions with like denominators (including mixed numbers)

Adds fractions with like denominators (including mixed numbers)

Demonstrates with objects the subtraction of fractions with like denominators (including mixed numbers)

Subtracts fractions with like denominators (including mixed numbers)

Category: Decimal Fractions

Demonstrates decimal fractions through thousandths

Reads and writes decimal fractions through thousandths

Writes decimal fractions for common fractions (denominators of 10, 100, and 1,000)

Adds and subtracts decimal fractions (through thousandths)

Topic: Other
Category: Estimation and Rounding

Rounds whole numbers to any given place

Rounds mixed numbers to the nearer whole number

Estimates products or quotients

Category: Exponents and Scientific Notation

Uses exponents to describe repeated factors

Converts exponential notation to whole numbers

LEVEL L

Topic: Whole Numbers
Category: Multiplication

Multiplies two-, three-, or four-place factors by two-place factors

Topic: Geometry and Measurement
Category: Time and Temperature

Names and records Celsius temperature (positive and negative)

Category: Length

Measures and records length, using decimal notation

Topic: Fractions
Category: Common Fractions

Identifies fractional names for whole numbers

Multiplies a whole number times a unit fraction

Category: Decimal Fractions

Writes common fractions for decimal fractions through thousandths

Orders decimal fractions through thousandths

Multiplies a whole number times a decimal fraction through thousandths

Multiplies a decimal fraction times a decimal fraction

Category: Ratio and Percent

Writes the ratio of two quantities as a fraction

Identifies a pair of equal ratios as a proportion

Topic: Other
Category: Number Sentences

Solves number sentences involving parentheses and more than one operation

Category: Estimation and Rounding

Rounds decimal fractions to the nearer whole number

Rounds decimal fractions to the nearer whole number, tenth, and hundredth

Category: Positive and Negative Numbers

Uses symbols for positive and negative numbers

Orders positive and negative numbers

LEVEL M

Topic: Geometry and Measurement
Category: Length

Changes to equivalent linear measures (milli through kilo)

Measures the radius, diameter, and circumference of a circle

Category: Weight and Mass

Relates the metric ton to the kilogram

Category: Area and Volume

Computes the area of a triangle

Computes the area of a quadrilateral (trapezoid, parallelogram)

Category: Geometric Figures

Constructs a circle and identifies its circumference, diameter, radius, and center

Distinguishes between lines and line segments

Describes lines: horizontal, vertical, parallel, and perpendicular

Distinguishes among quadrilaterals: rectangle, square, parallelogram, rhombus, and trapezoid

Category: Statistical Graphs and Tables

Determines scale for data

Constructs graphs for two variables

Category: Coordinate Graphs

Locates the point for given coordinates and names the coordinates of a given point in any of the four quadrants

Constructs graphs in four quadrants

Topic: Fractions
Category: Common Fractions

Demonstrates the meaning of multiplication for a unit fraction times a unit fraction

Multiplies fractions (less than 1)

Multiplies fractions (including mixed numbers)

Category: Ratio and Percent

Solves for a missing term in a proportion

Topic: Other
Category: Factors and Multiples

Names the least common multiple of two or more numbers to 16

Writes the prime factorization of any number through two places

LEVEL N

Topic: Whole Numbers
Category: Multiplication

Multiplies multiples of 100 or 1,000 by factors of three or more places (including multiples of 100, 1,000)

Multiplies factors of three or more places by factors of three or more places

Category: Division

Uses standard algorithm for dividing by two-place divisors

Topic: Geometry and Measurement
Category: Weight and Mass

Identifies the use of milligrams (mg)

Changes to equivalent metric measures (milli through kilo)

Category: Geometric Figures

Describes rays and angles

Identifies congruent angles by matching

Measures angles using a protractor

Constructs an angle of a given measure

Topic: Fractions
Category: Decimal Fractions

Divides a smaller whole number by a larger whole number or a decimal fraction through thousandths by a whole number

Writes decimal fractions for common fractions

Category: Ratio and Percent

Writes a ratio with a denominator of 100 as a percent (%) and a percent as a ratio with a denominator of 100

Topic: Other
Category: Number Sentences

Solves addition number sentences with positive and negative numbers

Category: Positive and Negative Numbers

Adds negative numbers

Adds positive and negative numbers

LEVEL O

Topic: Whole Numbers
Category: Numeration

Writes word names for numerals through four places

Writes numerals through billions

Category: Place Value

Expands decimal fractions

Expands numerals using exponents (whole numbers)

Category: Division

Divides by three or more place divisors

Topic: Geometry and Measurement
Category: Length

Computes the circumference of a circle

Category: Geometric Figures

Identifies and uses notation for a point, segment, ray, line, and angle

Distinguishes among open, closed, simple, and nonsimple curves

Distinguishes between concave and convex polygons

Distinguishes among interior, exterior, and curve itself

Determines the number of diagonals of a polygon

Identifies similar and congruent shapes

Topic: Fractions
Category: Decimal Fractions

Divides a whole number by a decimal fraction

Divides a decimal fraction by a decimal fraction

Divides decimal fractions (including mixed decimal fractions)

Topic: Other
Category: Number Sentences

Uses combinations of operations with whole numbers

Solves one-step number sentences, using decimal fractions

Category: Estimation and Rounding

Estimates products and quotients, using very large or very small decimals, common fractions, or whole numbers

LEVEL P

Topic: Geometry and Measurement
Category: Capacity

Estimates and verifies capacities by measuring in liters and milliliters

Changes to equivalent metric measures (milli through kilo)

Category: Geometric Figures

Uses a given scale to interpret a scale drawing

Constructs a scale drawing, given dimensions

Distinguishes among acute, right, obtuse, and straight angles

Identifies complementary, supplementary, vertical, and adjacent angles

Determines lines of symmetry in a triangle

Classifies triangles by comparing lengths of the sides

Classifies triangles by angle measure

Category: Statistical Graphs and Tables

Interprets scale for a pictograph

Interprets information from statistical graphs (pictograph, bar graph, line graph, circle graph)

Constructs a bar graph, using percent

Constructs a circle graph, using fractions

Category: Coordinate Graphs

Graphs the solution set of a mathematical sentence containing one variable

Topic: Fractions
Category: Common Fractions

Names equivalent fractions with a given denominator

Names least common denominators (LCD)

Adds common fractions with unlike denominators

Subtracts common fractions with unlike denominators

Multiplies three or more common fractions

Divides common fractions

Uses a combination of operations with common fractions

Category: Decimal Fractions

Reads and writes decimal fractions less than thousandths

Uses a combination of operations with decimal fractions

Category: Ratio and Percent

Writes a decimal (less than 1) as a percent and a percent (less than 100%) as a decimal

Writes a percent as a fraction in simplest form

Writes a common fraction as a percent

Finds a percent (greater than 1% but less than 100%) of a number

Topic: Other
Category: Number Sentences

Names the solution set for an equation

Solves one-step number sentences, using common fractions

Writes the mathematical sentence for a word sentence

Category: Factors and Multiples

Names all factors of a number

Applies rules of divisibility for 2, 3, 5, and 10

Writes the prime factorization of a number using exponents

LEVEL Q

Topic: Geometry and Measurement
Category: Capacity

States the relationship of a liter to a cubic decimeter to a kilogram

Category: Area and Volume

Computes the area of a circle

Computes the surface area of a right prism

Computes the surface area of a right circular cylinder

Finds and records the volume of a rectangular prism by counting cubic units

Computes the volume of a right prism

Uses exponential symbols for metric volume (cm^3)

Computes the volume of a right circular cylinder

Category: Geometric Figures

Demonstrates translations, reflections, rotations, and dilations

Identifies angles formed by lines intersected by a transversal

Identifies congruent angles formed by parallel lines intersected by a transversal

Distinguishes among drawings of solid shapes

Uses a straightedge and compass to copy a line segment

Uses a straightedge and compass to copy an angle

Uses a straightedge and compass to bisect a line segment and an angle

Uses a straightedge and compass to construct a perpendicular to a line through a point on the line

Uses a straightedge and compass to construct a perpendicular to a line through a point not on the line

Uses a straightedge and compass to construct a line parallel to a given line through a point not on the line

Category: Statistical Graphs and Tables

Constructs a circle graph, using percent

Constructs a circle graph, using measurement data

Constructs a frequency table

Category: Coordinate Graphs

Graphs inequalities, using decimal and common fractions

Topic: Fractions
Category: Common Fractions

Classifies fractions as terminating or repeating

Category: Ratio and Percent

Writes a decimal (greater than 1) as a percent and a percent (greater than 100%) as a decimal

Writes a decimal (less than .01) as a percent and a percent (less than 1%) as a decimal

Finds a percent (less than 1% or greater than 100%) of a number

Uses a proportion to find the percent one number is of another number

Uses a proportion to find the number of which a given number is a given percent

Topic: Other
Category: Number Sentences

Writes the mathematical sentence for a verbal inequality

Determines the solution set for an inequality

Category: Exponents and Scientific Notation

Simplifies the product of two or more numbers in exponential form

Simplifies the quotient of two numbers in exponential form

Writes the power of a power in exponential notation

Category: Positive and Negative Numbers

Names additive inverses for positive and negative numbers

Names equivalent forms for positive and negative numbers

Multiplies positive and negative numbers

Names reciprocals for positive and negative numbers

Divides positive and negative numbers

Adds positive and negative numbers

Subtracts positive and negative numbers

Simplifies expressions using two or more operations and positive and negative numbers

LEVEL R

Topic: Geometry and Measurement
Category: Statistical Graphs and Tables

Finds mean, median, and mode

Category: Coordinate Graphs

Graphs the solution set of a number sentence, using positive and negative numbers

Distinguishes among axes, origin, quadrants, and ordered pairs

Topic: Fractions
Category: Ratio and Percent

Writes probabilities of events as ratios

Topic: Other
Category: Number Sentences

Solves equations and inequalities containing positive and negative numbers and one variable by addition

Solves equations containing positive and negative numbers and one variable by multiplication

Solves equations containing positive and negative numbers and one variable by addition and multiplication

Category: Positive and Negative Numbers

Raises positive and negative numbers to powers

Changes scientific notation to customary notation

Writes a given number, using scientific notation

Finds the square roots of perfect squares

Approximates the square roots of numbers which are not perfect squares

Uses a table to name the square and square root of a number

Algebra 1 — Grade 8
Prerequisite: Attainment of the pre-algebra objectives in Elementary and Pre-Algebra Mathematics Objectives, Bulletin 285
3099 1 year no credit

Emphasis in Algebra 1 is on the development of an understanding of the basic structure of algebra related to the real number system; a recognition of the techniques of algebra as structures of this system; facility in applying algebraic concepts and skills; perception of the role of deductive reasoning in algebra; and an appreciation of the need for precision of language.

As techniques are developed, applications to appropriate, relevant problems are made; e.g., problems involving practical geometry, number theory, weather, air navigation, and money are included. Opportunities for enriching work in greater depth occur throughout the program.

Upon completion of Algebra 1, the student should be able to:

- demonstrate the use of the roster, rule, and graphing methods in representing sets
- compute the value of a numerical expression involving symbols of inclusion and the order of operations rules
- add, subtract, multiply, and divide directed numbers
- apply properties of equality and inequality and the concepts of additive and multiplicative inverses to solve equations and inequalities
- apply the fundamental operations to solve open sentences and word problems involving real numbers
- graph the solution sets of linear equations and inequalities
- add, subtract, multiply, and divide polynomials
- factor polynomials completely
- solve equations involving polynomials
- solve word problems involving the factoring of polynomials
- rewrite a linear equation into slope-intercept forms, and graph its solution set
- determine an equation of a line
- solve a system of open sentences in two variables
- solve word problems involving open sentences in two variables
- add, subtract, multiply, and divide algebraic fractions
- write radical expressions in simplest form
- solve quadratic equations
- solve equations and word problems involving irrational numbers

Unified Mathematics

Unified Mathematics sequence is offered by some schools as an alternative program for the student demonstrating exceptional mathematical and verbal abilities. Area office personnel assist elementary schools which feed the junior high schools offering this program to identify sixth graders who will be able to profit by enrolling in the program. Entry into the program is at the beginning of Grade 7 only.

Unified Mathematics is a contemporary, unified program developed at Teachers College, Columbia Univeristy. The unification of the traditional branches of mathematics is achieved by using the basic concepts of set, relation, mapping, and operation to study the basic structures of all mathematics — groups, rings, fields, and ultimately, vector spaces. These abstract *concepts* and *structures* are developed in coordination with the important realizations of these structures — number systems, geometries, probability, and later, analysis.

Unified Mathematics, Course 1 — Grade 7
Prerequisite: Attainment of designated pre-unified objectives in Elementary and Pre-Algebra Objectives, Bulletin 285
3017 1 year no credit

Course 1 begins with an investigation of finite number systems, comparing and contrasting their properties with those of the familiar set of whole numbers. Ideas and topics studied in this year include binary operation on a set; group properties; relations and mappings; the binary operation of composition; variables, expressions, equality, inequality, solution sets of open sentences; integers and rational numbers as mathematical structures; lattice points and figures in a plane; fundamental transformations; an introduction to probability, to number theory, and to applications.

Upon completion of Unified Math, Course 1, the student should be able to:

- compare properties of basic mathematical systems
- recognize a relationship between fundamentals of algebra and geometry through basic number theory and transformations
- apply various number systems to compositions of functions
- extend basic computational skills through operations with rational numbers
- identify algebraic examples of groups
- compute and demonstrate concepts of simple probability
- distinguish among various forms of transformations, and construct models for each

Unified Mathematics, Course 2 — Grade 8
Prerequisite: Attainment of the objectives of Unified Mathematics, Course 1
3018 1 year no credit

The concepts of set, relation, and mapping are treated more formally than in Course 1. Use is made of all the ideas learned informally in Course 1, developing these ideas with increasing depth and understanding. On this base, the first steps of the fundamental algebraic structures — group, ring, field — are constructed. The abstract structures are developed in coordination with the important realizations of these structures, especially those of sets, the real numbers, affine geometry, and coordinate geometry. Activities such as operations, solutions of sentences, uses of statistics, and graphical representations are presented in contemporary language and notation. Course 2 takes on a more formal, rational, and abstract form than Course 1.

Upon completion of Unified Math, Course 2, the student should be able to:

- apply techniques of linear and angular measure
- apply the formulas for area and volume
- construct logical arguments that result in valid conclusions, using various strategies of proof
- identify algebraic examples of fields
- apply algebraic and geometric techniques in solving linear and quadratic equations and inequalities
- apply properties of groups and fields to algebraic realizations

- extend basic computational skills through operations with real numbers
- construct formal geometric proofs using axioms and basic theorems
- determine measures of central tendency and variation for a set of numerical data
- interpret and apply Σ (sigma) notation
- solve problems using operations, graphs, and properties of real functions

MCPS INSTRUCTIONAL GUIDES — MATHEMATICS K-8

Elementary and Pre-Algebra Mathematics Objectives, Bulletin No. 285, 1975.
•*Elementary and Pre-Algebra Mathematics Instructional Guide*, 1978.

•This document is made available to teachers as their schools enter the Instructional System in Mathematics.